Propagation and Localization of Collective Excitations on a 24-Qubit Superconducting Processor

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Superconducting circuits have emerged as a powerful platform of quantum simulation, especially for emulating the dynamics of quantum many-body systems, because of their tunable interaction, long coherence time, and high-precision control. Here in experiments, we construct a Bose-Hubbard ladder with a ladder array of 20 qubits on a 24-qubit superconducting processor. We investigate theoretically and demonstrate experimentally the dynamics of single- and double-excitation states with distinct behaviors, indicating the uniqueness of the Bose-Hubbard ladder. We observe the linear propagation of photons in the single-excitation case, satisfying the Lieb-Robinson bounds. The double-excitation state, initially placed at the edge, localizes; while placed in the bulk, it splits into two single-excitation modes spreading linearly toward two boundaries, respectively. Remarkably, these phenomena, studied both theoretically and numerically as unique properties of the Bose-Hubbard ladder, are represented coherently by pairs of controllable qubits in experiments. Our results show that collective excitations, as a single mode, are not free. This work paves the way to simulation of exotic logic particles by subtly encoding physical qubits and exploration of rich physics by superconducting circuits.

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Mott insulators [34], and quantum walks [35–37]. Different from the 1D case, the Bose-Hubbard ladder can show unique emergent effects, especially, topological effects, which have been studied both theoretically and experimentally [38–40]. However, the dynamics of the Bose-Hubbard ladder model are not studied extensively. Thus, we would like to ask whether there are any special dynamical properties in the ladder model, which are distinct to the 1D case.

In this Letter, we experimentally demonstrate a Bose-Hubbard ladder, using 20 qubits on two legs of a 24-qubit superconducting processor, and investigate the dynamics of the single- and double-excitation modes. For the single-excitation case, we find it similar to a 1D chain, displaying the linear propagation [35–37] limited by Lieb-Robinson bounds. For the double-excitation case, we observe the localization of boundary collective excitations, which is a unique property of the Bose-Hubbard ladder. However, we find that the double-excitation mode is not stable in the bulk; instead, it splits into two single-excitation modes spreading linearly towards both sides, respectively. These phenomena show that the Bose-Hubbard ladder is not equivalent to the free-fermion system in the large-\(U\) regime, which is distinct to the 1D Bose-Hubbard chain. They will also help us further understand the mechanism and the nonequilibrium behaviors of strongly correlated many-body systems. The experimental observation of the distinctions between bulk and edge depends on that the ladder should be long enough. Our 20-qubit ladder of superconducting device satisfies this condition, and the boundary effects can be clearly identified. Furthermore, our experimental platform enables us to explore many other interesting topics in quantum computation and quantum simulations in different structures of lattices.

Our superconducting circuits, shown schematically in Fig. 1(a), can be described by the Bose-Hubbard ladder with a Hamiltonian (\(\hbar = 1\))

\[
H = \sum_{j\nu} J_{j\nu} (\hat{a}_{j\nu}^\dagger \hat{a}_{(j+1)\nu} + \text{H.c.}) + \sum_{j} J_{Rj} (\hat{a}_{jA}^\dagger \hat{a}_{jB} + \text{H.c.}) + \frac{U}{2} \sum_{j\nu} \hat{n}_{j\nu} (\hat{n}_{j\nu} - 1) + \sum_{j\nu} h_{j\nu} \hat{n}_{j\nu},
\]

FIG. 1. Experimental setup. (a) Circuit diagram of the device. There are 24 superconducting transmon qubits (\(Q_{1A}-Q_{10A}, Q_{1B}-Q_{10B}\) and \(Q_a-Q_d\)) constituting a ladder [41–45]. The nearest-neighbor hopping is realized by the capacitive coupling between qubits, and the negative anharmonicity \(U\) of each qubit gives the on-site attractive interaction. The frequency of each qubit is tunable by individual microwave driving through the flux-bias line. Readout resonators are separated into four groups, and the resonators in each group share a microwave transmission line for multiplexed readout. More experimental details of our system are presented in the Supplemental Material [46]. In our experiments, the middle 20 qubits (\(Q_{1A}-Q_{10A}, Q_{1B}-Q_{10B}\)) are biased to the working frequency 4.8 GHz. The other four qubits (\(Q_a-Q_d\)) are always idled with frequencies \(< 3.5, 3.5, 5.4, \text{and } < 3\) GHz, respectively. Thus, \(Q_a-Q_d\) are far away from the working frequency, and thus, a 20-qubit ladder is employed. (b) The image of the corresponding Bose-Hubbard ladder with 6 rungs. The red ball represents the photon (the excitation of the qubit), which can tunnel to the nearest-neighbor sites (red arrows). We label two qubits, which share the same rung (black box), as a unit cell \(C_j\).
where $j$ denotes the number of rung, $\nu = A/B$ denotes the leg $A/B$, $\hat{a}_{j\mu}^\dagger$ ($\hat{a}_{j\mu}$) is the bosonic creation (annihilation) operator, $\hat{n}_{j\mu} \equiv \hat{a}_{j\mu}^\dagger \hat{a}_{j\mu}$ is the number operator, $h_{j\nu}$ is the tunable local field strength, set to be the same during the quench dynamics, $U$ is the on-site interaction, $J_{j\mu}$ is the nearest-neighbor coupling strength of the leg $\nu$, and $J_{jR}$ is the coupling strength of two qubits at the rung $j$. The excitation in the superconducting qubit can be regarded as a photon in the microwave regime [16,35]. For convenience, we regard two sites, which share the same rung, i.e., $Q_{jA}$ and $Q_{jB}$, as a unit cell labeled as $C_j$, with $j = 1, \ldots, 10$. Then, this system can also be taken as a quasi-1D model, see Fig. 1(b).

In our superconducting circuits, the nearest-neighbor hopping strength $J_{j\mu}/2\pi \approx J_{jR}/2\pi \approx 12$ MHz, and the anharmonicity $U/2\pi \approx -230$ MHz. Because $|U|/J_{j\mu} \approx 19 \gg 1$, two photons can hardly bunch at the same site, indicating that the photons in this system behave as hard-core bosons. Therefore, the state of $C_j$ can be approximatively expanded by four basis $|00\rangle_j$, $|01\rangle_j \equiv \hat{a}_{jB}^\dagger|00\rangle_j$, $|10\rangle_j \equiv \hat{a}_{jA}^\dagger|00\rangle_j$, and $|11\rangle_j \equiv \hat{a}_{jA}^\dagger \hat{a}_{jB}^\dagger|00\rangle_j$, where $|00\rangle_j$ is the vacuum state, representing that there is no photon at $C_j$. Here, we regard $|01\rangle_j$ and $|10\rangle_j$ as single-excitation states, and $|11\rangle_j$ as the double-excitation state. In this case, the nonlinear term of $H$ in Eq. (1) vanishes, and the system reduces to a single-particle problem. Thus, with $J_{jA} \approx J_{jB}$ assumed, we can write an effective Hamiltonian of $H$ as [46]

$$H_{\text{eff}} = \sum_{j=1}^{10} J_j (\hat{a}_{jA}^\dagger \hat{a}_{jA+1} + \hat{a}_{jB}^\dagger \hat{a}_{jB+1} + \text{H.c.}) + \sum_{j=1}^{10} \mu_j (\hat{a}_{jA}^\dagger \hat{a}_{jA} - \hat{a}_{jB}^\dagger \hat{a}_{jB}),$$

where $\hat{a}_j \equiv (\hat{a}_{jA} + \hat{a}_{jB})/\sqrt{2}$, $\hat{f}_j \equiv (\hat{a}_{jA} - \hat{a}_{jB})/\sqrt{2}$, $\tilde{J}_j \equiv (J_{jA} + J_{jB})/2$, and $\mu_j \equiv J_{jR}$. Then, we can find that there are two decoupled free modes in the system. Moreover, note that with the qubit representation, $\hat{a}_j^\dagger |00\rangle_j = (|10\rangle_j + |01\rangle_j)/\sqrt{2}$ and $\hat{a}_j^\dagger |01\rangle_j = (|10\rangle_j - |01\rangle_j)/\sqrt{2}$ are triplet state and singlet state, respectively.

First, we investigate the quench dynamics of the system with single-excitation initial states. During the time evolution, we measure the probability distribution of the single-excitation state, $P_{j1}^\text{exp} \equiv P_0^j + P_1^j$, by simultaneous single-shot readouts of two qubits at $C_j$, where $P_0^j$ and $P_1^j$ are the probabilities of single-excitation states $|01\rangle_j$ and $|10\rangle_j$ of $C_j$, respectively. In Figs. 2(a) and 2(b), we prepare a single-excitation state $|10\rangle_j^1$ localized at the left edge ($C_1$) and $|10\rangle_j^6$ at the central of the bulk ($C_6$), by exciting the corresponding qubits after the system’s initialization. In Fig. 2(c), we prepare a state $|10\rangle_j^1 \otimes |01\rangle_j^6$ with two single excitations localized at both edges ($C_1$ and $C_6$).

For these three cases, we observe the linear propagation of single excitations. In Figs. 2(d)–2(f), we compare the corresponding ideally theoretical predictions with experimental data in Figs. 2(a)–2(c). The experimental results agree well with the theoretical predictions.
In the single-particle representation, the effective Hamiltonian in Eq. (2) describes two decoupled 1D free hardcore-boson systems. These two free modes are both linear combinations of two single-excitation states, corresponding to the Bell states of two qubits. Hence, the propagation of a single excitation is linear and limited by the Lieb-Robinson bounds \[2\] with the maximal group speed \[v_{\text{max}} = 145.9 \text{ cells/μs} \] \[46,47\], see Fig. 2(g). To extract the group propagation velocity \(v_g\) of the single excitation, we use the linear fitting between the time of the first probability peak and the cell distance, as shown in Fig. 2(h). The experimental group velocity is \(v_{\text{ex}} = 112.4 \text{ cells/μs}\), and the theoretical one is \(v_{\text{th}} = 123.5 \text{ cells/μs}\) by using device parameters, between which the difference may result from unbalanced qubits’ frequencies during the experiments.

Then, we study the dynamics of double excitations, starting with only one double-excitation initial state \([11]_j\) at \(C_j\) after system initialization. In Figs. 3(a) and 3(b), theoretical and experimental evolutions of the probability distributions of double-excitation states \(P^{11}_j\) for qubit-qubit unit cells are shown, where the initial double-excitation state is localized at the leftmost cell (\(C_1\)). By analyzing the distributions \(P^{11}_j\), we can find that the evolution of the double-excitation state displays boundary localization, which is quite distinct from the case of the single-excitation states. When the initial double-excitation state is in the bulk, e.g., at the central cell (\(C_6\)) as shown in Figs. 3(c) and 3(d), we can observe no localization. Instead, this double-excitation state splits into two independent single-excitation modes, spreading linearly toward two edges, respectively, see Figs. 3(e) and 3(f), which shows photons’ antibunching.

In our system, since the large-\(U\) condition is satisfied, the photons can exhibit fermionization and behave like free fermions, implying negligible double occupancy at a single qubit. However, the edge localization of the double-excitation state cannot be described by free fermions \[46\]. In contrast to the single-excitation case, the dynamics of the double-excitation states cannot be described by the single-particle representation, and the correlation of two modes neglected in Eq. (2) should be considered. In fact, in the large-\(U\) limit, the Bose-Hubbard model more likely conforms to the XX spin systems. In the Supplemental Material \[46\], we present a phenomenological interpretation of the double-excitation dynamics in the perspective of the XX spin ladder.

Then, to see how the on-site interaction strength \(U\) affects the edge state localization, we present in Fig. 4 the numerical results of the dynamics of the ideal
Bose-Hubbard ladder ($J_{JA} = J_{JB} = J_{JR} = J$) for different regimes of interaction strength. We find that the edge localization of double-excitation states can only exist in the large-$U$ regime. The edge localization disappears in the small-$U$ regime, because of large probabilities of bosons’ bunching at the same sites [46], which can further show that the edge localization results from the interactions.

In addition, we study the correlation functions of the ground states of the XX spin ladder by the density matrix renormalization group (DMRG) methods [46]. The simulation results clearly reveal that for the double excitations, their edge correlations decay much faster than that of the bulk, and this different scaling behavior shows that the ground state also has the similar boundary effects. This is a phenomenon, absent from free-fermion systems but rooted in the interaction effect, and our 20-qubit experimental system can really capture it.

In conclusion, we have theoretically and experimentally studied the quench dynamics of the Bose-Hubbard ladder on the superconducting circuits. Benefiting from the precise control and readout of a 24-qubit superconducting quantum processor, we directly observe (i) the linear propagation of single excitations, (ii) the localization of the edge double excitations, and (iii) the instability of bulk double excitations with a 20-qubit ladder. Our experiments show that collective excitations are not free on the Bose-Hubbard ladder, which cannot be described in the single-particle picture. Our results may be useful for further studies on the statistical properties of particles in strongly correlated systems. Moreover, in addition to recent work on the Bose-Hubbard chains [35], our platform provides the possibilities for further explorations of the distinctions between chain and ladder models, e.g., magnon scattering, the scaling of correlation functions, and entanglement entropies [48–50].

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