Temperature Oscillations in a Compartmentalized Bidisperse Granular Gas

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A granular clock is observed in a vertically vibrated compartmentalized granular gas composed of two types of grains with the same size. The dynamics of the clock is studied in terms of an unstable evaporation or condensation model for the granular gas. In this model, the temperatures of the two types of grains are considered to be different, and they are functions of the composition of the gas. Oscillations in the system are driven by the asymmetric collisions properties between the two types of grains. Both our experiments and model show that the transition of the system from a homogeneous state to an oscillatory state is via a Hopf bifurcation.

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At first sight, granular gases [1–3] share many similarities with their molecular counterparts. However, for compartmentalized systems, phenomena which are impossible for equilibrium molecular gases such as Maxwell's demon [4] or clustering [5] can occur in granular gases. These phenomena are possible because differences in granular temperatures [6] can be stably sustained between the compartments. Recently, an even more intriguing phenomenon known as the granular clock [7,8] has been reported. For the clock, the clustering of the grains between the compartments becomes periodic (Fig. 1) when the monodisperse gas is replaced by a bidisperse one.

The mechanism of granular clocks is still unclear. One possible origin is the Brazil nut effect (BNE) or the reverse BNE (RBNE) when there is vertical segregation [7,8]. Since the clustering in monodisperse gas is driven by the difference in temperatures between the compartments, it is plausible that the oscillation in temperatures in a bidisperse system is also a driving force. For a bidisperse system, it is still not clear if there is only one single temperature [9] for the whole system or if the two types of grains have their own temperatures [10,11]. Intuitively, the scenario of a single temperature in a bidisperse gas would not lead to clustering oscillation because it is similar to the case of monodisperse gas. We are then left with the possibility of dual temperatures in the system. Presumably, the clock is driven by the oscillations of the dual temperatures.

In this Letter, we report results of an experiment designed to remove the effect of BNE in a compartmentalized system by using a bidisperse system with grains of the same size. We find that a granular clock can still be produced. Our results can be understood by a model developed to understand the mechanism of the granular clock in terms of dual temperatures. No BNE or RBNE assumptions are needed. Therefore, even grains of the same size in a bidisperse system can produce granular oscillations. Predictions of our model are verified by experiments. Both our experiments and model show that the transition from a homogeneous state to an oscillatory state in such a system is via a Hopf bifurcation.

Our experimental setup consisted of a rectangular glass container 2.6 cm wide and 5.4 cm long with a height of 13.3 cm, divided into two equal compartments by an aluminum barrier 0.2 cm thick and with a height (h) of 1.5 cm from the bottom. Steel balls (SBs) and glass beads (GBs) of the same size, 0.5 mm in radius, are used. In the experiments, the total number of grains is fixed at 960,

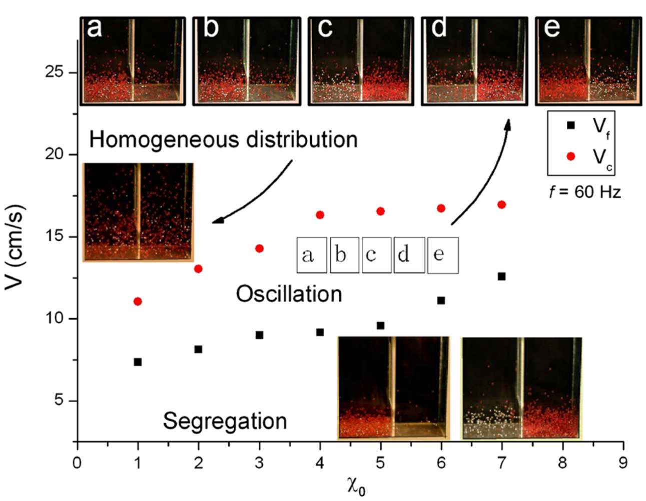

FIG. 1 (color online). The phase diagram for the homogenous (HS), oscillatory (OS), and segregated (SS) states. See the captions of Figs. 3 and 4 for the uncertainties in v_c and v_f. Also shown are the photographs of the configurations of the system with glass beads being darker (red) and steel balls being lighter (silver). Snapshots (a)–(e) are taken during one cycle of the clock. Note that $f = 60$ Hz and the period of the clock (Fig. 4) can range from tens to hundreds of seconds.

while the ratio $\chi_0 = N_{\mathrm{GB}}/N_{\mathrm{SB}}$ varies from 1:1 to 7:1, where N_{GB} and N_{SB} are the number of glass beads and steel balls, respectively. Vertical sinusoidal vibrations with frequency f and amplitude a are applied to the container to create granular gases with a shaking velocity v with $v = 2\pi af$.

Results of the experiments are summarized as a phase diagram in terms of v and χ_0 as shown in Fig. 1. Similar to the findings of Ref. [8], three states [homogenous (HS), oscillatory (OS), and segregated (SS)] in the distribution of grains can be observed. These phases are separated by two transition velocities v_f and v_c which are functions of χ_0. Experiments with different sets of a and f have shown that the boundaries of these three regimes are sensitive only to the combination of $af \propto v$. For a fixed value of χ_0, as the v is being lowered, the system is first in HS when $v > v_c$ and then turns into OS when $v_c > v > v_f$. The system becomes segregated when v is lowered below v_f. The features of the transition points v_c and v_f are that the amplitude of the oscillations (Δ) increases with $(v_c - v)$ while the period of the oscillation (τ) decreases with $(v - v_f)$. Note that Δ is close to saturation around v_f, and it is impossible to determine τ close to v_c because of strong fluctuations in the system. Obviously a new type of granular clock is produced without the BNE or RBNE as shown in the snapshots for the OS in Fig. 1.

Let us first consider a two-compartment system which is made up of a right compartment (RC) and a left compartment (LC). Hereafter, quantities in the LC and RC will be denoted by the L and R subscripts, respectively. If the granular clock experiments were performed with molecular gases, one would have needed to change the temperatures (T) of the two compartments in a periodic manner externally. For example, a molecular gas can be made to evaporate from LC and condense into RC if T_L is raised higher than T_R and vice versa. We show below that this periodic temperature change can be generated endogenously by an instability in the dynamics of a bidisperse granular gas.

For a monodisperse granular gas, there is a single granular temperature for the system which is approximately $T \propto (\frac{u}{N})^2$, with N being the number of grains and u is some characteristic velocity which is related to the shaking velocity (v) of the system [4]. The approximation of a single height-independent temperature has been shown to be rather satisfactory by molecular dynamics simulations [4,5]. T is inversely proportional to N^2 because most of the dissipation of the system comes from the collisions of the grains among themselves, and higher density means more collisions and therefore a lower T. When v is large, one would expect that the grains are free to move between LC and RC. Therefore, the grains are distributed equally ($N_R = N_L = N/2$) in LC and RC with $T_R = T_L$. However, when v is lowered to a point that most of the grains in RC and LC cannot be exchanged, the system can be unstable. Consider a fluctuation of ΔN in the number of grains in the RC, i.e., $N_R = N/2 + \Delta N$, $N_L = N/2 - \Delta N$. In this case, T_L will be higher than T_R, and more grains will jump from LC to RC than those from RC to LC. During this process, T_L is raised further by the evaporation of grains and vice versa for the condensation in RC. Therefore this process is unstable, and it will not stop until the difference in grain number between the two compartments is so large that the smaller probability for grains to jump from RC to LC is compensated by a large number of N_R. This is the scenario in Ref. [4].

Next, consider the temperature in a bidisperse granular gas in a single compartment with a total number of $N = N_A + N_B$ grains and a number ratio $\chi_0 = N_A/N_B$ between the two types of grains A and B. Although a single T is sometimes assigned for such a system [5], we argue that T_A and T_B are in general different. For such a system, there are three types of binary collisions: namely, AA, AB, and BB. The most important one here is the AB collision which is the source of energy exchange between the two types of grains. Let us further assume that during AB collisions, B is slowed down by A while A is accelerated by B. That is, A is getting kinetic energy from B. For such a system, T_A and T_B will be a function of χ_0. For example, T_A will be higher when χ_0 is smaller and vice versa for T_B. In other words, the temperature of B grains is lowered by the presence of A while the presence of B grains increases the temperature of A grains. Therefore, when χ changes, the temperatures for a particular kind of grain will also be changed.

For such a bidisperse gas in a two-compartment system, relevant parameters are $N_A = N_{AL} + N_{AR}$, $N_B = N_{BL} + N_{BR}$, $\chi_L = \frac{N_{AL}}{N_{BL}}$, and $\chi_R = \frac{N_{AR}}{N_{BR}}$. For high v, we will have $N_{AL}/N_A = N_{BL}/N_B = \frac{1}{2}$ (i.e., $\chi_L = \chi_R = \chi_0$), $T_{AL} = T_{AR}$, and $T_{BL} = T_{BR}$ with $T_B < T_A$. When the strength of shaking is lowered, we can always get to a point at which only type A grains are free to exchange between the two compartments. When this happens, the system will be similar to that of a single type of A. In such a case, our arguments about clustering will be applicable. That is, there will be an unstable evaporation and condensation of A. However, the situation here is more complicated because there will be changes in χ_L and χ_R during this process.

When grains A evaporate from RC and condense into LC, both T_{AL} and T_{BL} are lowered while those in RC will be raised. When enough of grains A are evaporated from RC, the T_{BR} will be raised, and it can be so high that grains B start to evaporate from RC and condense in LC, too. Since this is an unstable situation, once it happens, more of grains B will jump from RC to LC. However, when grains B start to jump from RC to LC, T_{AL} starts to rise, too. When enough B has jumped from RC to LC, T_{AL} will be so high that grains A start to jump from LC back to RC. Again, this is an unstable situation which will lead most of grain A to jump from LC to RC. That is, there is an oscillation in grains A. Similar arguments will show that there will also be oscillations in grains B, too.

Let us denote the masses and radii of grains A and B by m_A, m_B and r_A, r_B, respectively, the restitution coefficients of AA, BB, and AB collisions by e_A, e_B, and e_{AB}, respectively, and the bottom area of the compartment by Ω. With these notations, we have $T_{AL} = (\frac{v_A}{D_A N_{AL}})^2$ and $T_{BL} = (\frac{v_B}{D_B N_{BL}})^2$, where $D_A \equiv 4\sqrt{\pi} r_A^2 (1 - e_A^2)/\Omega$ [12] and similarly for D_B. The two velocities v_A and v_B are related to v as $v_A = v/p(\chi)$ and $v_B = v/q(\chi)$ for some general functions p and q, where $\partial p/\partial N_{BL} < 0$ and $\partial q/\partial N_{AL} > 0$. The requirements on $\partial p/\partial N_{BL}$ and $\partial q/\partial N_{AL}$ are used to implement our assumption that the temperature of grains A is raised by the presence of B, while the presence of grains A lowers the temperature of grains B. It can be shown [13] theoretically that the situation of grains A gaining energy due to A-B collisions occurs when $e_{AB} m_B > m_A$. With this formulation, there are two equations for the balance of the fluxes of A and B between RC and LC as

$$\begin{aligned} \frac{1}{K_A}\dot{N}_{AL} &= -\frac{N_{AL}^2}{v_A(\chi_L)} e^{-a_A[N_{AL}^2/v_A^2(\chi_L)]} \\ &\quad + \frac{(N_A - N_{AL})^2}{v_A(\chi_R)} e^{-a_A[(N_A-N_{AL})^2/v_A^2(\chi_R)]} \\ \frac{1}{K_B}\dot{N}_{BL} &= -\frac{N_{BL}^2}{v_B(\chi_L)} e^{-a_B[N_{BL}^2/v_B^2(\chi_L)]} \\ &\quad + \frac{(N_B - N_{BL})^2}{v_B(\chi_R)} e^{-a_B[(N_B-N_{BL})^2/v_B^2(\chi_R)]}, \end{aligned} \tag{1}$$

where $a_A = ghD_A^2$ (a_B is similarly defined) with g being the acceleration due to gravity and h is the height of the wall separating LC and RC; K_A (K_B) are constants that depend on the properties of A (B). Note that the forms of the above equations are similar to those used in Refs. [7,8]. However, the physical meanings of the above equations are different from those of Refs. [7,8]. Here the dynamical variables are the number of grains A and grains B in one of the compartments, whereas those of Refs. [7,8] are the *fractions* of grains A and B in one of the compartments. For grains of a single type [4], the *fraction* completely determines the properties of the system. But for a binary mixture, one also needs χ to specify the effects of one type of grain on the other.

Obviously, $N_{AL} = N_A/2$ and $N_{BL} = N_B/2$ is a fixed point, but it will be stable only when v is large. When v is smaller than a threshold v_c, it will become unstable and turn into a limit cycle. To be specific, we use reasonable functional forms $p(\chi) = \frac{1}{2}(2 - \frac{1}{1+\chi})$ and $q = 2p$ (form I) for actual calculations. Note that $v_A = v$ when $\chi \to \infty$ and $v_B = v$ when $\chi \to 0$ as expected. Furthermore, it can be shown that [14] the fixed point loses its stability at v_c via a supercritical Hopf bifurcation giving rise to a stable limit cycle. We have also checked that the same behavior holds for other reasonable forms such as $p(\chi) = 1 - \frac{1}{2}e^{-\chi/\chi_0}$, $q(\chi) = 1 + \chi/\chi_0$ (form II), and $p(\chi) = 1 - \frac{1}{2}e^{-[(\chi/\chi_0)^2/2]}$, $q(\chi) = 1 + (\chi/\chi_0)^2$ (form III).

Figure 2 shows the null clines of Eq. (1) for different values of v together with the trajectory of an arbitrary initial point. In Fig. 2, we have chosen the values of a_A and a_B in Eq. (1) such that v_c is close to unity for a system with $K_A = K_B = 1$, $N = 2000$, and $\chi_0 = 2$. Note that the difference in a_A and a_B can originate from either the difference in the restitution coefficients or in their radii. It can be seen that the fixed point $(\frac{N_A}{2}, \frac{N_B}{2})$ [Fig. 2(a)] turns into a limit cycle when v is lowered [Fig. 2(b)]. As v is further reduced, both the amplitude (Δ) and period (τ) of the limit cycle increases [Fig. 2(c)]. When Δ is large enough, new stable fixed points are created [Fig. 2(d)]. The main difference between our model and that of Ref. [8] is that in Ref. [8] BNE is needed while only asymmetry in collision properties is needed here, be it mass or size.

In the experiments, it can be observed that the SB is giving energy to the GB during collisions and therefore the GB is playing the role of grains A. To compare the model with the experimental results quantitatively, we examine Δ and τ of the oscillation close to the fixed points v_c and v_f. In the experiments, τ is measured by a stopwatch and Δ is measured by counting the number of steel SBs in a compartment by stopping the experiment when the SBs are seen to be least populated in one of the compartments. Figure 3 is the measured dependence of Δ on $v - v_c$ together with a fit of the form $\Delta \sim (v_c - v)^{1/2}$. It can be

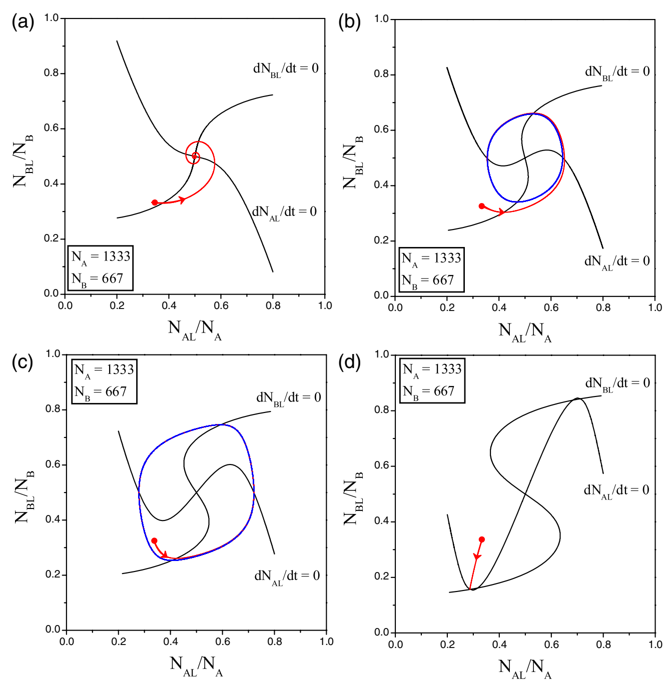

FIG. 2 (color online). Simulated phase trajectories as a function of shaking velocity v showing the three different states of the systems. (a) Stable fixed point for the homogeneous state at $N_{AL}/N_A = N_{BL}/N_B = 1/2$ with $v = 1.01$, (b) the oscillatory state as a limit cycle when $v = 0.99$, (c) the amplitude Δ of the limit cycle increases when v is further lowered to 0.97, and (d) finally another fixed point is reached where there is a stable segregation of grains with $v = 0.927$.

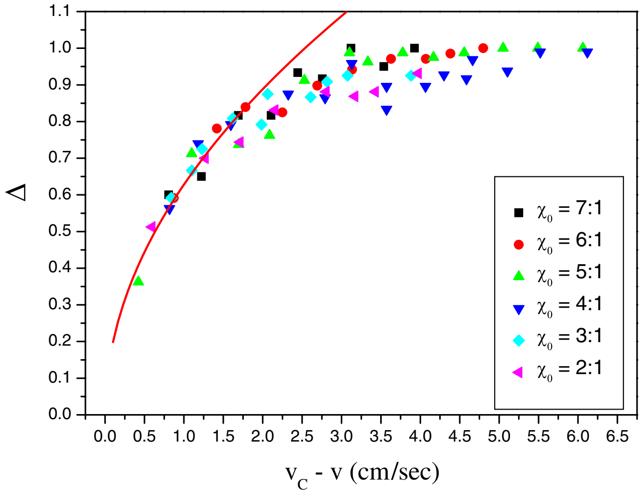

FIG. 3 (color online). Variation of oscillation amplitude Δ as a function of distance away from the critical velocity $(v_c - v)$ for various χ_0. The solid line is a fit of the data to the form $\Delta \sim (v_c - v)^{1/2}$ for small $v_c - v$ with v_c being adjusted within 20% of the experimental observation to give the best fit. Uncertainties of the data are smaller than the size of the symbols.

seen that our data are consistent with a supercritical Hopf bifurcation only for small $v_c - v$. Presumably, this is due to the fact that linear stability analysis will fail when $v_c - v$ is large. Figure 4 shows both the $(v - v_f)$ dependence of τ obtained from experiments and from our model from Eq. (1) (inset). It can be seen that all of the data can be put in the form $\tau \sim (v - v_f)^{-\alpha}$ with $\alpha \simeq 1$ for the experiments and different values of α for different forms of p and q.

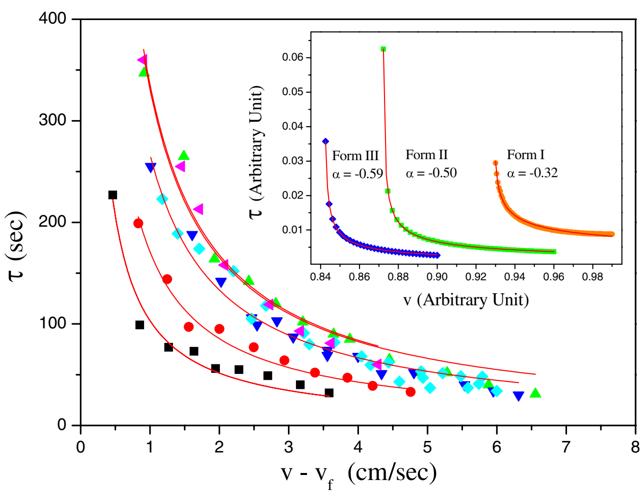

FIG. 4 (color online). Variation of τ as a function of $v - v_f$ measured for various χ_0. The symbols are the same as those in Fig. 3. The solid lines are fits of the data to the functional form $\tau \sim (v - v_f)^{-1}$ with v_f being a fitting parameter. The inset shows the results from the calculations of Eq. (1) with the three different forms of p and q. Note that the calculated results can all be fit to the form $\tau \sim (v - v_f)^{-\alpha}$ but with different values of α. Uncertainties of the data are smaller than the size of the symbols.

This last finding suggests that our flux model gives a good quantitative description of the physics of the system only close to the unstable point, independent of the forms of p and q used. However, detailed dynamics of the system will be needed to determine the value of α.

It is clear that our model captures the essential features of the clock phenomenon. In fact, our argument can be applied even to the heaping phenomenon [15]. For such a case, in the language of our model, grains from the majority parts of the compartment are evaporated and then condensed at one of the corners of the compartment to form the heap. There will be large temperature differences between the grains in the heap and those outside the heap. Finally, it is known that the RBNE can also be driven by density differences [16]. However, our system is a dilute gas; the effects considered in Ref. [16] are probably not applicable.

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