Topological Charge Pumping in a One-Dimensional Optical Lattice

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A topological charge pump transfers charge in a quantized fashion. The quantization is stable against the detailed form of the pumping protocols and external noises and shares the same topological origin as the quantum Hall effect. We propose an experimental setup to realize topological charge pumping of cold fermionic atoms in a one-dimensional optical lattice. The quantization of the pumped charge is confirmed by first-principles simulations of the dynamics of uniform and trapped systems. Quantum effects are shown to be crucial for the topological protection of the charge quantization. Finite-temperature and nonadiabatic effects on the experimental observables are discussed. The realization of such a topological charge pump serves as a firm step toward exploring topological states and nonequilibrium dynamics using cold atoms.

Introduction.—Charge pumping is a standard method to generate a steady current in solid-state circuits [1–5] through adiabatically and periodically time-varying potentials. The effect bears a similarity to the famous Archimedes screw [6], where water is pumped by a rotating spiral tube. However, quantum physics offers a more intriguing phenomenon: quantum charge pumping, where the charge transferred in each pumping cycle is exactly quantized. Thouless [1] has shown that the one-dimensional (1D) quantum charge pump shares the same topological origin as the two-dimensional (2D) quantum Hall effect (QHE) [7]. The amount of pumped charge can be expressed by the Chern number of a 2D QHE Hamiltonian [8]. In other areas of condensed matter physics, the theory of quantized charge pumping also lays a firm foundation for the modern theory of polarization of crystalline solids [9,10], the theory for Z2 spin pump [11,12], and inspired the theoretical connection [13] between the 3D Z2 topological insulators and the 4D quantum Hall effect [14]. The word quantum in the quantum charge pumping has twofold meanings. First, the pumped charge is quantized. Second, one actually relies on the quantum mechanics (thus, the concept of Berry phase and energy gap) for the topological protection of the quantized charge.

A clean and highly tunable cold atom system provides an opportunity to realize and detect this topological charge pumping effect. Specifically, advances in constructing optical superlattice structure [15–17] and nonequilibrium control of lattice intensity and phases [18] allow the realization of a charge pumping setup, which we will propose in this Letter. In situ detection with the single-site resolution [19–21] allows the detection of topological charge pumping. The equivalence of 1D topological charge pumping and the 2D quantum Hall effect connects our proposal to recent efforts of exploring topological quantum phases with synthetic gauge field [22–25] and spin-orbit couplings [26–28], where one of the landmarks is to realize the quantum Hall effect [7] and topological insulator [29] state in atomic quantum gases.

In this Letter, we consider the topological charge pumping of cold fermions in a 1D optical lattice potential. First, we show that the proposed potential indeed realizes the topological charge pumping by calculating its Chern number and ab initio simulation of the pumping process. Compared with the corresponding classical dynamics, we show that quantum effects are crucial for the topological protection of the quantized charge pumping. We then consider the effect of a harmonic trap and predict the topological quantization of the center of mass of the cloud in realistic experimental situations.

Our proposal is based on a time-dependent 1D optical superlattice of the form [15–18]

\[ V_{OL}(x, t) = V_1 \cos^2 \left( \frac{2\pi x}{d} \right) + V_2 \cos^2 \left( \frac{\pi x}{d} - \varphi(t) \right). \]  (1)

We use the lattice constant d as the unit of length and the recoil energy \( E_R = (h^2 \pi^2 / 2md^2) \) as the unit of energy, where \( m \) is the mass of the atom. Such superlattices have been experimentally realized in Refs. [15–18]. The lattice strengths \( V_1 \) and \( V_2 \) and the phase factor \( \varphi \) can be tuned dynamically.

We propose to vary the relative phase linearly with time

\[ \varphi(t) = \frac{\pi t}{T}. \]  (2)

The lattice then changes in time with a period \( T \). In the absence of the static short wavelength lattice controlled by the \( V_1 \) term, the \( V_2 \) term describes a sliding lattice shown in Fig. 1(a). Including the \( V_1 \) term, one realizes the Rice-Mele model [30] in a continuous space setup, as illustrated in Figs. 1(b) and 1(c). This can easily be seen by expanding the \( V_2 \) term (neglecting spatially independent constants)
Multiband effects are fully included in our calculations [34]. A continuum model also allows us to directly compare with classical pumping dynamics and to demonstrate the importance of quantum effects for the topological protection of the pumped charge.

Figure 2(a) shows the spatial-temporal structure of \( V_{OL}(x,t) \) for \( V_1 = V_2 = 4E_R \), which we will refer to as the Rice-Mele pumping potential in the following. Performing a Fourier transform of \( H(x,t) \), we obtain the Bloch Hamiltonian \( H(k_x,t) \), which satisfies the periodicity conditions \( H(k_x+2\pi,t) = H(k_x,t) \) and \( H(k_x,t+T) = H(k_x,t) \). Since there is always a gap to higher bands, we can calculate the Chern number of the lowest band of \( H(k_x,t) \) as if it was a two-dimensional Hamiltonian [35,36]. Figure 2(b) shows the Berry curvature distribution in the \( k_x - t \) space. Integration of the Berry curvature over the Brillouin zone shows that the Chern number is equal to 1.

Thouless showed [1] that at zero temperature under the adiabatic approximation, the pumped charge equals the Chern number for a filled band. We now proceed to simulate the pumping process and directly calculate the pumped charge. For an infinite system, the pumped charge is defined through the integration of the total current (see the Supplemental Material for details [37]):

\[
\text{Pumped charge} = \int C \, dt
\]

\( C \) being the total current. Figure 2(c) shows the pumped charge for different values of the parameters. Integration over the Brillouin zone shows the Chern number equals 1. (c) Pumped charge and total current Eq. (5) of an infinite-sized system with \( T = 40\hbar/E_R \) and one particle per unit cell. The pumped charge is quantized at full pumping cycles.

FIG. 2 (color online). (a) Spatial-temporal structure of the optical lattice Eq. (1) for \( V_1 = V_2 = 4E_R \). (b) Berry curvature distribution of the lowest band of \( H(k_x,t) \). Integration over the Brillouin zone shows the Chern number equals 1. (c) Pumped charge and total current Eq. (5) of an infinite-sized system with \( T = 40\hbar/E_R \) and one particle per unit cell. The pumped charge is quantized at full pumping cycles.
\[ \Delta n(t) = \int_0^t \, dt' J(t'). \] (5)

Figure 2(c) shows the current and pumped charge for pumping cycle \( T = 40\hbar/E_R \) in the Rice-Mele pumping potential (the band gap is \( \sim 1.5E_R \)). The sudden onset of pumping causes high frequency oscillations of the current. The pumped charge is quantized at times that are multiples of the cycle time \( T \). Our calculations show that modifying the quantum pump by changing \( V_1 \) and \( V_2 \) results in a different current \( J(t) \); however, the pumped charge remains quantized. One thus realizes a topological pump, neither relying on a tight-binding approximation nor the details of the pumping protocol.

Figure 3 shows nonadiabatic and finite-temperature effects on the quantization of pumped charge. Quantization is precise for slow pumping and low temperature compared to the band gap. Considering \(^{40}\)K atoms and \( d = 532 \text{ nm} \), one has \( \hbar/E_R = 36.4 \mu  \text{s} \) and \( E_R/k_B = 0.21 \mu \text{ K} \). Thus, for a pumping period longer than \( 50\hbar/E_R = 2 \text{ ms} \) and initial temperature lower than \( 0.1E_R/k_B = 20 \text{ nK} \), the pumped charge is quantized to within 0.2%. Such a pump is feasible within current experimental abilities.

To demonstrate the importance of quantum mechanics for topological protection, we examine classical pumping in the same lattice potential. For a sliding lattice (\( V_1 = 0, V_2 = 4E_R \)), both the quantum and classical pump transfer units charge in one cycle. However, mapping the classical problem to a classical pendulum (see the Supplemental Material [37]) shows that this is accidental, and the pumped charge is not exactly quantized. This accidental quantization is removed by changing the atom mass, the lattice constant, or the pumping potential. Figure 3 shows that the pumped charge drops to close to zero for the classical Rice-Mele pump at low temperature. It is because of the potential minima felt by the classical particle does not shift in space. On the contrary, the quantization in the quantum case is protected by an energy gap and survives as one distorts the pumping potential to the Rice-Mele model. The difference between classical and quantum behavior is due to the absence of Berry phases and energy gaps in classical dynamics. This comparison highlights the importance of quantum effects for topological protection.

At finite temperatures, quantization in the quantum pump remains stable for temperature smaller than the energy gap (see Fig. 3). The quantum to classical transition is determined by the condition \( n\lambda \ll 1 \), where \( \lambda = \hbar \sqrt{2\pi\beta/m} \) is the thermal de Broglie wavelength, \( \beta \) is the inverse temperature, and \( n \) is the average density of the system. Classical behavior dominates when \( \beta^{-1} \gg (4(nd)^2/\pi)E_R \). For a shallow optical lattice \( nd \sim 1 \), the quantum to classical crossover happens at temperatures much larger than \( E_R \). For \(^{40}\)K atoms and \( d = 532 \text{ nm} \), the whole temperature region of the quantum to classical crossover in Fig. 3 can be achieved in experiments.

**Trapped system.**—To connect to real experimental situations, we link the quantization to a simple physical observable: the center of mass of a cloud in a harmonic trap \( V_{\text{trap}}(x) = (1/2)m\omega_x^2x^2 \), which varies slowly compared to the optical lattice. In Fig. 4(a), we show the initial

![FIG. 3 (color online).](image-url) The pumped charge after one cycle vs temperature for the quantum and classical case. Quantization of the pumped charge is visible for temperature lower than the band gap. For the classical case, there is no such topological protection and the pumped charge depends on the pumping protocols. The inset shows the nonadiabatic (finite-pumping time) effect on the pumped charge of a quantum Rice-Mele pump. The upper axis shows the realistic temperature and time estimated for \(^{40}\)K atoms in a \( d = 532 \text{ nm} \) laser.

![FIG. 4 (color online).](image-url) (a) Initial density distribution in a harmonic trap with \( \omega_x = 0.03E_R \), \( V_1 = V_2 = 4E_R \), and particle number \( N = 40 \). The green curve shows the continuous space density, while the blue curve shows the occupation number integrated over each unit cell. (b) Occupation number after several cycles of pumping with \( T = 40\hbar/E_R \); time increases from left to right. The cloud shifts to the right, and the center of mass position is quantized; see Fig. 5.
pumped charge and c.m. at every full pumping cycle. The similarities between Figs. 2 and 5 show that the external trap and finite size of the atomic cloud do not affect the precise quantization of the pumped charge. The observation of this effect in cold atom systems is thus highly feasible, with in situ imaging techniques for the atomic cloud [19–21].

Finite-size effects and metallic edges will, in principle, give a nonquantized value of the pumped charge. Any such deviation from an integer value is, however, not visible in our simulations and will be even smaller in the experimental situation where the trap is larger and finite-size effects are thus smaller. While topological pumping is stable against weak interactions [40], interaction effects can also easily be avoided by using spin-polarized atoms.

Conclusion.—We have proposed a realistic experimental setup to realize the topological pumping of cold atoms. Our setup naturally interpolates between sliding potentials and the Rice-Mele model [30] commonly studied in condensed matter physics. The quantization of the pumped charge can be observed from a quantization of the center of mass motion of the atomic cloud, is independent of the details of the pumping protocol, and is robust with respect to nonzero temperature. The experimental observation of topological pumping in cold atoms will be a big step toward exploring topological states and nonequilibrium dynamics in cold atom systems.

As further steps, interactions on a fractionally occupied lattice may open up an energy gap, and one could pump a fractional charge in each pumping cycle. With two spin species, it will be interesting to see $Z_2$ spin pumping where the Wannier centers of two time-reversal-symmetrical states split and exchange [12,41].

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Figure 5 shows the total current $J$, pumped charge $\Delta n$, and c.m. $\langle x \rangle$ in a trap. The relationship Eq. (8) is evident from the plot, and we clearly see quantization of the pumped charge and c.m. at every full pumping cycle. The similarities between Figs. 2 and 5 show that the external trap and finite size of the atomic cloud do not affect the precise quantization of the pumped charge. The observation of this effect in cold atom systems is thus highly feasible, with in situ imaging techniques for the atomic cloud [19–21].

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[32] This equivalence is only limited to the lowest band since the band gap between higher bands closes.


